MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/2022

DEM5056 – CALCULUS

(DEE & DME)

9 AUGUST 2022
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages (excluding the cover page and **1** page of appendix).
2. Answer **ALL** questions. All necessary working steps must be shown.
3. Write all your answers in the answer booklet provided.

QUESTION 1 [17.5 MARKS]

a. A function $f(x)$ is defined as: $$f(x)=\begin{cases} 2x & x<-1 \\ -x^2+4 & -1\le x<2 \\ 5 & x=2 \\ -0.5x+2 & x>2 \end{cases}$$

Determine the values of $f(-2)$, $f(-1)$, $f(0)$, $f(2)$ and $f(3)$. [5 marks]

b. Consider the following functions: $f(x)=\sqrt{x+11}-4$, $g(x)=x-5$

Evaluate the following limits:

i. $\lim_{x\to -11} f(x)$ [2 marks]

ii. $\lim_{x\to 5} g(x)$ [1.5 marks]

iii. $\lim_{x\to 5} \frac{f(x)}{g(x)}$ [4 marks]

iv. $\lim_{x\to \infty} \frac{g(x)}{f(x)}$ [2 marks]

c. Consider the following functions: $r(x)=2x^2-4x$, $s(x)=x^2+2x-8$

Evaluate $\lim_{x\to -4} \frac{r(x)}{s(x)}$ [3 marks]

QUESTION 2 [32.5 MARKS]

a. Determine the derivative of the following functions.

i. $y=x^{1.99}+\ln(e)$ [2 marks]

ii. $y=5^x+e^{5x}$ [2.5 marks]

iii. $y=\cos^3(4x^2+1)$ [5 marks]

iv. $y=\log_{10}(9x)+\ln(\ln(2x))$ [4.5 marks]

Continued...

b. For the function $y=(5x+4)^2$, do the following:

i. Use Chain Rule to evaluate the $\frac{dy}{dx}$. [1.5 marks]

ii. Use Product Rule to evaluate the $\frac{dy}{dx}$. [2 marks]

(Hint: Rearrange the function in the form that is suitable to apply Product Rule)

c. Consider the function, $P(x)=x^4+2x^3+4$. Determine the following:

i. The local extreme(s). [8.5 marks]

ii. Inflection point(s) if $P''(x)=12x^2+12x$. [4 marks]

iii. The concavity interval of the function. [2.5 marks]

QUESTION 3 [35 MARKS]

a. Using suitable integration method, evaluate the following integrals.

i. $\int \frac{x}{x^3+4x}dx$ [3.5 marks]

ii. $\int 3xe^{2x}dx$ [7 marks]

iii. $\int_0^1 x\sqrt{9-x^2}\,dx$ [7 marks]

b. Find the area of the shaded region in Figure 1 and 2 using integration method.

i.

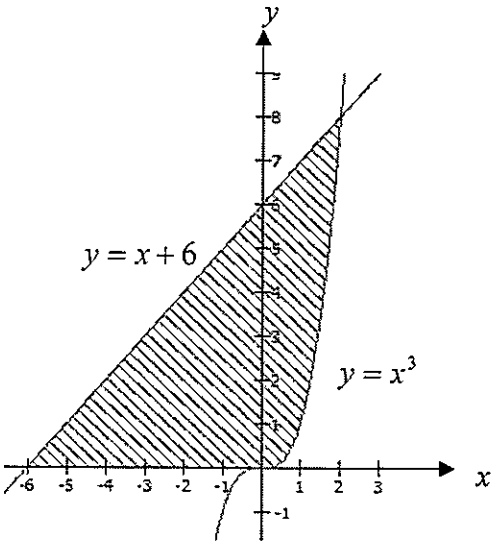

Figure 1 [5 marks]

Continued...

ii.

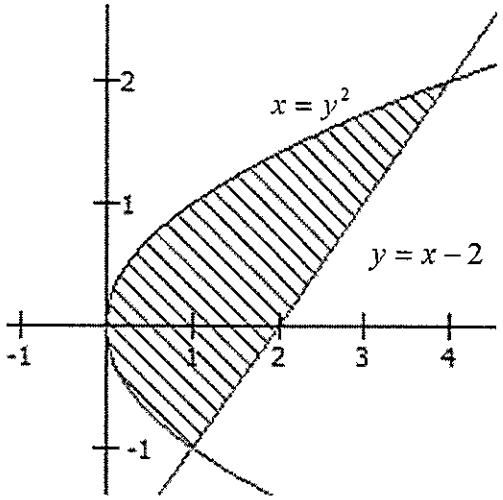

Figure 2 [5 marks]

c. Find the volume of solid generated when the shaded region in Figure 3 below is rotated about y–axis.

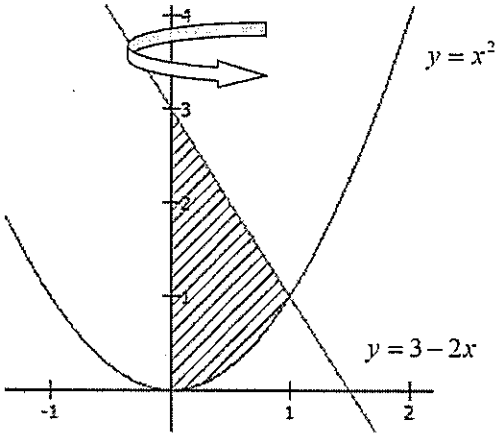

Figure 3 [7.5 marks]

QUESTION 4 [15 MARKS]

a. Use the Chain Rule to find $\frac{\partial w}{\partial t}$ if $w = x^2 + y^2 + z^2$, $x = e^t \cos t$, $y = e^t \sin t$ and $z = e^t$. [5 marks]

b. A lamina that occupies a region between $x = 0$, $x = 4$, $y = 0$ and $y = 3$ has a density of $\rho(x, y) = x^2 y$.

i. Find its mass. [2 marks]

ii. Find its center of mass. [8 marks]

End of page.

APPENDIX I: Formulae

Integration of common functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C \qquad \int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C \qquad \int \csc x \cot x dx = -\csc x + C \qquad \int \sec x \tan x dx = \sec x + C$$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx \qquad A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx \qquad V = \int_c^d \pi\left(\left[w(y)\right]^2 - \left[v(y)\right]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx \qquad V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Triple Integrals:

$$\iiint_B f(x,y,z) dV = \int_r^s \int_c^d \int_a^b f(x,y,z) dx dy dz$$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a, b) is a critical point of f]. Let

$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.

b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum

c. If $D < 0$, then $f(a, b)$ is a saddle point.

Moments and Centers of Mass

The **moment** about the ***x*-axis:**

$$M_x = \iint_D y\rho(x,y) dA$$

The **moment** about the **y-axis:**

$$M_y = \iint_D x\rho(x,y) dA$$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\rho(x,y) dA \qquad \bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\rho(x,y) dA$$

Where the mass:

$$m = \iint_D \rho(x,y) dA$$